

On Philosophical and Empirical Aesthetics

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Abstract

Aestheticians have generally not followed the example of other philosophers (such as those concerned with theories of mind) in assimilating recent scientific findings, among which those in the experimental psychology of perception and cognition, and in empirical (psycho-) aesthetics, are especially relevant. Can aesthetics afford to remain aloof from empirical findings on relevant phenomena and from scholarly thought that is backed by experiment? Should aestheticians' talk of "new aesthetics" risk being isolated from the most promising scientific trends? Or is it time to begin to profit from the fruits of the scientific method and to admit the despised (or dreaded) empirical findings into the generous bosom of aesthetics? The present paper will specifically try to show how the discussion of two well-publicized topics — the "transhuman stance" in aesthetics (Wolfgang Welsch) and "beautifying beauty" (Sasaki Ken-ichi) — could have amply profited from an awareness of contemporary psychology of perception and cognition and of empirical aesthetics. It will be argued that a revolutionary change is needed in how aesthetic knowledge is acquired and that a scientific, empirical, aesthetics will introduce rigor and a greater conceptual discipline into aesthetic discourse.

Key words: empirical aesthetics, psycho-aesthetics, science and philosophy, music psychology, beauty.

Introduction: Baumgarten, Kant, Zeising, Fechner

In his *Critiques*, one of Kant's principal objectives was to analyze how certain human cognitive activities were possible. Traditionally, in philosophy, the first, *Critique of Pure Reason*, is thought successfully to answer the question of "how is

knowledge of nature possible" (Owen & Strong, in press). Given the purpose of this question, a scientist who is disappointed that not the slightest substantive advance in the knowledge of the facts of nature follows in the book is simply being naive about philosophical inquiry. However, Kant's answer -- that the conditions necessary for the knowledge of nature to occur must be "transcendental" -- disappoints even an epistemologist's more realistic expectations. After all, it is known that Kant read Newton and von Leibniz and was thus well aware of the assumptions, logic, and methodology of the experimental approach -- in short, the intellectual and practical "conditions" which made the giant steps in the knowing of nature possible. To put it bluntly, Newton and the scientists who followed him have had no need of Kant, whereas Kant and the philosophers who followed him have been, at best, astute commentators on science.

It is also instructive to remember the words by which Kant, in the Critique of Pure Reason, dismissed Alexander Baumgarten's approach to aesthetics. According to Kant, Baumgarten ("that admirable thinker") tried to subsume "the beautiful under rational principles, and to raise its rules to the rank of science" (Davey, 1992, p. 41). This, to Kant, was undesirable precisely because the rules are "empirical and...can never serve as determinate a priori laws by which our judgment of taste must be directed" (Kant, 1781/1970, p. 66).

In view of this, it is interesting to contemplate how Kant would respond to the axiomatic, "top-down," approach being challenged not by "empirical rules," but in practice, for example, by Zeising's careful measurements of the "golden section" in the human body and in sculpture (Zeising, 1854, 1855, 1884), and, especially, Fechner's (1871, 1876) "aesthetics from below," which included laboratory and field experiments, archival measurement of paintings, and important and systematic methodological discoveries (cf. Konečni, 1997, 2004b).

Gustav Fechner can therefore be considered as one of the co-fathers of *empirical* aesthetics and he, obviously because of the invaluable knowledge gained already during his lifetime in sensory psychophysics (another area which he co-fathered), gave this new, "bottom-up," aesthetics a distinctly psychological flavor -- so much so that much of empirical aesthetics can properly be termed "psychological aesthetics" or "psycho-aesthetics." Psycho-aesthetics should be regarded as a branch of experimental psychology, which studies normative and accurately measured sensory, emotional, and cognitive human responses (including judgments) to objectively defined and accurately scaled stimuli. The methodology and logic of inquiry in psycho-aesthetics are those of scientific psychology, not of philosophy or philosophical aesthetics.

Empirical Verification of Aesthetic Issues

The main thesis of the present essay is that to the extent that most, if not all, questions of aesthetics are empirically verifiable, the field needs the scientific method to advance; at the very least, the scientific methodology that has been developed in psycho-aesthetics is needed to introduce rigor and discipline into the discourse of philosophical aesthetics, which is often characterized by "authoritative" assertions that are backed by little more than anecdotes and (occasionally) interesting literary narrative. Unfortunately, in contemporary philosophical aesthetics, the anti-empirical stance often equals, quite simply, poor scholarship.

It will not do to say, for example, that philosophical aesthetics is akin to theoretical physics and that the field of aesthetics needs a non-empirical branch. Theoretical and experimental physics are fully integrated by both mathematics and the assumption of desirability of experimental verification. The tools of mathematics and the logic of experimentation make the theoretical and experimental branches of physics the results of a coherent and systematic division of labor.

Music Theory as a Good Example?

Music theory appears at first to be a far better example, one that might be marshaled by philosophical aestheticians as a field that is self-contained, fully developed, and successfully non-experimental. However, such claims for music theory are actually inaccurate and therefore represent a misleading, illusory defense for the continuing pre-scientific and non-experimental status of aesthetics. It is not only that music theory deals with an elegant, complete, rule-based language, which is a far cry, even in its extreme post-modern form, from the chaotic babble of "art" and other topics of philosophical aesthetics. Rather, the point is that music theory itself is self-contained only insofar as it limits itself to being a didactic tool for composers and performers. Once the listener (I would claim necessarily) is admitted into the picture - and, importantly, regardless of the level of his or her musical expertise, so long as the issue is actually listening to music, not one of consulting the score -- the problems become profoundly psychological. They are the problems of auditory perception, and of the reception and organization of sound, which can, and have been, studied by the methods of experimental psycho-aesthetics.

In such studies, many a dogma of traditional music theory has been overturned. These include: Redefinitions of the effects and significance of macro- and micro-structure in some of the pinnacles of Western music; tonal closure; the relative effects of pitch and spatial proximity; the awareness of repetition; the perceptual significance of Schönbergian operations on the tone row in serial music; and many others (e.g., Cook, 1987, 1990; Deutsch, 1984; Konečni, 1984; Konečni & Karno, 1994; Tillmann & Bigand, 1994).

These experimental findings were at first angrily challenged from the position of armchair authority by music theorists; then, some of them timidly proposed "more appropriate" experiments and musical materials; then, some of them learned to do experiments, or at least to collaborate with psycho-aestheticians and psychologists of music. The outcome was a markedly different awareness of the issues of perception and reception of music and a radically changed discourse in music theory. Note that these "concessions" were extracted by *experiments* -- and that these were carried out because of the researchers' conviction that there existed a logical possibility that certain music-theoretical claims can be empirically verified (cf. Batt, 1987; Gotlieb & Konečni, 1985; Karno & Konečni, 1992; Konečni, 1987).

None of these advances could have been accomplished by even an astute "philosopher of music," such as, for example, Peter Kivy, because he and others limit themselves to speculative, impressionistic, anecdotal commentary and *ad hoc*, often simplistic, categorizations of phenomena. Kivy's multi-oeuvre efforts appear informative, but are, in the big picture, mere segments of a futile debate that is based on assertions by (pseudo-) authority (Konečni, 2003).

Two Illustrative Topics

In the remainder of this essay, the focus will be on two topics: The "transhuman stance" in aesthetics, advocated by Wolfgang Welsch (2001), and the notion of "beautifying beauty," discussed by Sasaki Ken-ichi (2001). The discussion will center specifically on how these papers — highly influential and prominently discussed by (philosophical) aestheticians since the 15th International Congress of Aesthetics in Tokyo, 2001 — could have profited from at least an awareness of, if not close familiarity with, the contemporary psychology of perception and cognition, and of empirical aesthetics.

The "Transhuman Stance"

The first sentence of Welsch's "Summary" states: "Though art is certainly made by humans, it often tends to disguise its human provenance and to present instead something not reducible to a human scale." When one mentally runs a film of the key stages of art production from forty thousand years ago at Lascaux to now, "often" is hardly the qualifier that comes to mind for describing the frequency of concealment or disguise of art's "human provenance."

Instead of carefully describing, defining, and categorizing – the essence of the scientific method and the scholarly approach in general – Welsch resorts to examples. These come from West and East, and from painting, music, and horticulture. But instead of this variety resulting in clarity and the revelation of an underlying truth through the careful selection of examples and the principle of "triangulation," one has a mélange, the characteristics of which are verbally manipulated to support the "transhumanity" thesis, but could be made to support virtually any other. The essay is held together not by the logic and substance of the examples, but by the forced contrasts and dichotomies that attempt to avoid the all-too-obvious contradictions and non-sequiturs contained in the material.

Welsch's attempt to promote the idea of the "transhuman stance" as "one of art's intrinsic features" fails because the idea, while "cute," is fundamentally wrong. There is, however, a quite different common thread in Welsch's examples (some are listed below) -- one that he might have pursued to advantage had he been cognizant of well over a hundred years of work in experimental psychology on the visual and auditory illusions. The complex perceptual effects of Kasimir Malevich's Black Square, Jean Dubuffet's Gala de terre and Fruits de terre, Walter de Maria's The 2000 Sculpture, John Cage's Thirteen, the garden of the Nanzen-ji -- are all comprehensible in the light of the current and past experimental work.

There are by now even solid secondary sources, which should have precluded the nebulous statements and outlandish claims that Welsch (2001) makes: "Malevich tried to make us experience a cosmic dimension" (p. 5); "Dubuffet reveals a terrestrial transhuman world" (p. 6); (in reference to de Maria:) "When geometrical construction is practiced such that the result is, amazingly, beyond comprehensibility, then the modern belief in human mastery through geometry is shaken; through geometrical operation, an opening beyond the standard human world occurs" (p. 7); (quoting K.-i. Sasaki:) "The specific effect of the beauty of nature [...] is also to console us, by whispering to us that we are of the same substance" (footnote 31, p. 13; this neglects that Western cultures – from animism, to mythology, to Christian dogma, to evolutionary theory -- have always contained significant elements of human oneness with nature); "Mi Fu ... was a poet, painter, and critic (1051-1107) who felt at odds with average human behavior and

self-perception and instead worshipped the peculiar shape of rocks -- praising them as models of true human being" (p. 15; italics added); (a subtitle:) "Overcoming Western anthropocentrism -- on the way to a 'rational Buddhism' " (p. 20); (another subtitle:) "From inhuman to transhuman" (p. 21).

In summary, one defensible meaning of "transhumanity" in art is that art sometimes relies on perceptual illusions, which are subject to the scientifically established laws of perception like other perceptual phenomena and have been thoroughly investigated by experimental psychologists. Western artists have sometimes systematically studied perceptual phenomena (as was the case with perspective -- which, incidentally, has never been fully discovered or utilized in Far-Eastern art), sometimes stumbled on them without knowing the underlying scientific laws, and sometimes, as was the case especially in the 20th century ever since the inception of the Gestalt theory of perception by Wertheimer, Koffka, and Köller, collaborated closely with psychologists (beginning with the artists associated with the Bauhaus, De Stijl, and abstraction).

Fantasy is another aspect of the artistic process to which the "transhuman" may logically be taken to refer -- although its use here, as in the case of the perceptual illusions, is superfluous and obfuscates rather than clarifies. Artists indeed fantasize and invent and include dream-work in technically highly skilled depictions of the irrational, the surreal, the materially nonexistent, and the never-experienced by the senses. And although at a technically vastly inferior level, but with incredible flights of fancy and fantasy, my son, who is eight years old, is constantly dealing with the nonexistent and the "transhuman." It is redundant to manufacture concepts such as the "transhuman stance" in order to discuss the major role of fantasy in art and play -- which is, like the laws of perception, a subject of a truly enormous literature in developmental psychology and psycho-aesthetics.

Laws of perception, fantasy, and imagination are the logically and empirically defensible cornerstones to which Welsch would have to be referring if his talk of the "transhuman stance" and the "transcending tendency" is to make any sense. His terms are therefore unnecessary, bordering on obscurantism. They should be especially resisted because of the unsubtle evocation of Kant to lend authority to the idea and even more so because of the even less subtle "millennial" implication.

"Beautifying Beauty"

The strategy of the paper is laid out in the Summary (Sasaki, 2001, Footnote 9, p. 27). First, there is a strange oversimplification of the concept of the beautiful in Western art and aesthetics: "The modern Western aesthetics catches the beautiful in the following situation: I perceive the beautiful in ... an artwork. The beautiful is strictly placed in a spot." Then the incorrect simplification is given a label ("crystallization") and thus artificially fixed in the discourse as one pole of an arbitrary dichotomy. Then the (simple-minded?) West is contrasted with (the deep and ever-so-different?) Japan: "I find a radically different vision in a well-known waka (short poem) by Akiko Yosano (1878-1942): 'Traversing Giyon (sic) quarter in Kyoto to Kiyomizu-dera Temple, people I met this evening of cherry blossoms are all beautiful" (the 31-syllable Japanese original is given later in the text). Then an arbitrary interpretation of the poem is given ("the beauty of blossoms beautifies the world"), followed by a label ("radiation") to be contrasted with "crystallization." To conclude the Summary, there is the bombast with millennial overtones: "This vision

["radiation"] represents an aspect of the beautiful completely ignored in the modern world, which might illustrate what we need in the new century."

So one has here a straw position against which to draw a conceptual contrast of dubious validity; and one has arbitrary nomenclature, arbitrary typology (A and B only, which is typical of such typologies in quasi-science), and a huge unjustified leap of inference -- all of these based on a rather pedestrian image in one (mediocre?) poem by a regional celebrity. In fact, even the meager facts of the poem are misread in order to fit the arbitrary conclusion. Furthermore, biographical facts are marshaled (which is strange for a poem so modest in artistic content) to support the conclusion -- quite unconvincingly.

Thus, the attempt to develop a general idea (of "beautifying beauty") is based on a single waka by Yosano Akiko, which, Sasaki admits (p. 30), some waka specialists "rate...as worthless" (an opinion with which he disagrees, but I do not). A very general aesthetic idea indeed is developed from a Japanese poetic form, with the action taking place not just in Kyoto (the most Japanese of Japanese cities), but in its old Gion pleasure quarter, and not just at any time of the year, but in the cherry-blossom season. In other words, an enormous generalization is drawn from a very specific example. This is, of course, not to say that Sasaki's thought process could not have arrived at the generalization (however unjustified) after being set in motion by the poem, but in that case the poem should have occupied a marginal, rather than a central, place in the essay. No, Sasaki needs the poem because he needs the local, the traditional, the folkloric Japanese elements — a miniature from which to make the "giant leap." In other words, the strategy is itself a literary form and a literary effort. However, the endeavor is of very limited scholarly value.

In the writing, there is hyperbole and insistence of Japanese "specialness." Not just cherry blossoms but the moon itself is different in Japan than in Europe, according to Sasaki. But he knows the West (Schönberg's moon, and Dracula's).

To the extent that there is any conceptual mystery in the poem it is this: Does the moonlit evening in cherry-blossom season (the protagonist does not actually see the blossoms, for they are not in Gion, but in the nearby public park, Maruyama-kôen, so she may just as well be referring simply to a pleasant night in early April) cause the passersby to be beautiful (Causal Model 1: Direct effect of evening on people's appearance)?; or does the evening cause the protagonist to be happy and this happiness makes others appear beautiful (Causal Model 2: Effect of evening on people's appearance mediated by the protagonist's internal state, happiness)?; or is a another variable, say, a romantic appointment, responsible for the protagonist's happiness, with the thrill further intensified by the pleasant evening and things it evokes in the cherry-blossom season, so that the evening and the happiness jointly, but with unequal weights, make passersby appear happy (Causal Model 3: Romantic happiness would have been sufficient to make the passersby beautiful, but it is aided in its effect by the pleasant evening)?

This type of thinking, in terms of causal models and "path analysis," is common in psychology and other sciences. In well-designed experiments, one can measure with considerable confidence the correctness of the various alternatives. Even if the experiments are not carried out, the models formalize the logical structure of the elements and processes in the problem at hand. It takes Sasaki many pages to touch upon the elements mentioned in the preceding paragraph. There is conceptual

confusion, especially with regard to the subject and object, and the possibility of continuous feedback loops of information between the two. There are naïve mentions of "psychological" issues. What strikes one most while reading Sasaki's account is the arbitrariness and lack of rigor in the reasoning, in the marshaling of evidence (such as is marshaled), and in the process of inference.

Without, of course, designating them as such, Sasaki opts for Causal Model 1 and rejects a simplified version of Model 3 (See Sasaki in Footnote 9, p. 37: "I have interpreted the phrase 'Tonight all the people I pass are beautiful'...as describing how the radiation of the beauty of the cherry blossom beautifies the people, and rejected the reading that the people looked beautiful because [the protagonist, the poetess] Akiko was happy"). However, this is caprice, not a scholarly decision: I cannot see any value of this conclusion for aesthetic theory, nor should this level of scholarship allow anyone to make millennial pronouncements and suggest paradigmatic shifts.

Incidentally, it is interesting that a millennialist of a very different caliber, L. N. Tolstoy (who was perhaps not a "typical Westerner," but certainly not Japanese either) wrote something very pertinent to the present discussion. Of course, on this occasion, in *Anna Karenina*, Tolstoy (1877/1918/1995) did not bother with either aesthetic pseudo-theories nor with utopian morality, but, fully believing in the experiential, existential, and psychological correctness of my Causal Model 2, wrote this about Levin's perceptions of people and their behavior -- Levin, who had been made supremely happy by Kitty's declaration of love and consent to marry him:

"At the confectioner's, florist's, and jeweller's, he noticed that they were expecting him, that they were pleased to see him, and triumphed in his happiness just like everyone else with whom he had anything to do... It was extraordinary that not only was everybody fond of him, but all the hitherto unsympathetic, cold, or indifferent persons were delighted with him, gave way to him in everything, treated his feelings with delicate consideration, and shared his own opinion that he was the happiest man on earth because his betrothed was the height of perfection. Kitty felt just the same..." (p. 406).

So, the prospect of marriage to this particular person, Kitty, makes Levin happy and this happiness makes ordinary shop-keepers appear not just outwardly beautiful, but also inwardly kind. Had it been, say, a loveless, pre-arranged marriage, the shop-keepers would look as they did before. Not a "thing of the world" -- the fact of marriage, but the resultant internal state of this particular union, happiness, is what causes people's beautification in Tolstoy's (and my) opinion.

Analogously, an evening in early April in Gion, with cherry-blossoms nearby, would probably not be a powerful enough stimulus to make passersby appear beautiful to Akiko: It is more likely that her happiness, perhaps due to romance, produced the effect and the evening and the season only strengthened it.

But one cannot be sure. I have doubts. The problem is that Sasaki does not.

The point of the mental exercise above was not to prove that Sasaki's "theory" is wrong and Tolstoy's correct, but precisely to demonstrate the logical impossibility of the philosophical method to arrive at a logically conclusive inference. Experiments are needed to disentangle the causal-model alternatives and they, in fact, could be performed on the subject-matter in question.

However, it might be interesting to continue the mental exercise a little, because the relative correctness of the various models does have implications, including those for the expenditure of public funds, if one thinks of the models from the standpoint of environmental aesthetics (that Sasaki brings up in the final section of his essay). For example, if happiness, one's internal state, makes the world beautiful, there is no point for a city to spend funds on, say, public art when the local populace is hungry and miserable -- although, in accordance with Tolstoy's model, the locals appear beautiful to the *already* happy foreign tourists on honeymoon, the ones who really profit emotionally from the local art. This is obscene, but one can daily see examples in many unfortunate countries (Konečni, 2004a).

Whether or not the concept of "beautifying beauty" is correct or not, in terms of Yosano Akiko's waka or the "real world," East and West, it should be pointed out that the issues in question are profoundly psychological and have been investigated by experimental psychologists for some 140 years. Chief among them are classical (Pavlovian) conditioning, the roles of temporal and spatial contiguity in emotion and cognition, and the mutually related perceptual principles of assimilation and contrast. Artists are capable of understanding such concepts: Stanislavski used Pavlovian conditioning in his *method*, while Kandinsky and Mondrian, among others, knew and applied the Gestaltists' experiments on assimilation and contrast. The implications of "Pavlov's dogs" have reached even cartoonists, let alone textbooks, and there are erudite secondary sources (notably books by E. H. Gombrich) dealing with perceptual principles in art and emotion.

Such psychological concepts, and the results of experiments that had been carried out to investigate them, could have provided Sasaki with a firmer grasp of "beautifying beauty," as well as placed the ideas of "brilliance" and "aura" (that he half-heartedly considered) on a more secure footing.

"Idol of the Tribe"

Instead of worrying about, say, "anthropocentrism," contemporary aesthetics should worry about scholarly rigor. Even more, it should worry about being an irrelevant backwater of contemporary science -- not just science "in general," but those of its branches that deal with content that is highly relevant to aesthetics. Instead of contemplating the trans-human stance and the beautifying of beauty, aestheticians might seriously consider the implications of Francis Bacon's Novum Organum. In his thorough search for a new paradigm that would combine sensory data and speculative theory, Bacon dismissed both the restoration of Greco-Roman ideas and, especially, the anti-empirical stance of scholasticism. But, he thought, the biggest barriers to clear thought and progress in science (and, therefore, the condition of human life) lay within the four "idols" existing in the human mind. Of these, the most difficult to deal with, Bacon thought, was the "idol of the tribe," the inherited foibles of human judgment and thought.

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