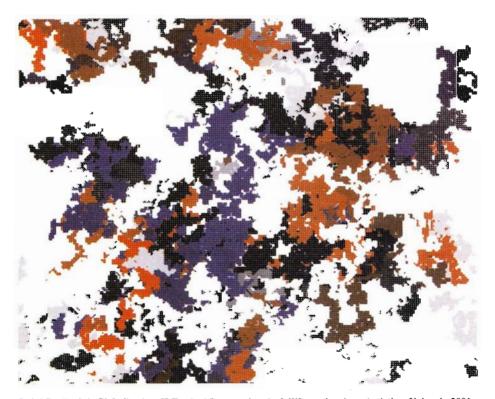
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THE GOLDEN SECTION IN THE STRUCTURE OF 20*-CENTURY PAINTINGS'

Abstract

The goal of this empirical study was to examine the respective uses of the golden section (0.62) and other proportions of theoretical interest. Structural elements within paintings, their vertical and horizontal balance, and their overall dimensions were expressed as ratios for the 95 members (obtained from an initial sample of 250) of a quasi-random sample representing all the decades of the 20th century. Whereas the respective functions of the golden section and other proportions were found to vary subtly as a function of the measurement target, the golden section was generally used to offset its more staid rivals 0.50 (in certain symmetrical relationships) and 1.00 (as in the circle and square) and introduce a degree of tension and complexity.

THE GOLDEN SECTION IN THE STRUCTURE OF 20*-CENTURY PAINTINGS

Proportions, along with balance (or the harmonious distribution of perceptual "weights" around some axis), are the key to an understanding of the composition of a work of art — presumably no less so in 20th-century painting.

Among the proportions, a special place has been occupied by the "golden section" This proportion ("divine," according to Johannes Kepler) that has been thought by many to be a universal principle in art and an important determinant of beauty, has been transmitted for some 2,600 years by mainly European elites — mathematicians, philosophers, monks, artists, architects, aestheticians (from Pythagoras to Euclid to Vitruvius to Fibonacci to Pacioli da Borgo to Kepler to Zeising to Fechner to Mondrian to Le Corbusier, to give just a short list of admirers). The elegance of the arithmetic, geometric, and even biological contexts in which the golden section is found is undisputed, yet the significance of the ratio in the creation, perception, and enjoyment of art has been the subject of almost furious debate (e.g., Berlyne, 1971; Borissavlievitch, 1958; Boselie, 1992; Bouleau, 1963; Godkewitsch, 1974; Hoege, 1997; Le Corbusier, 1954).

The basic example of the golden section is the division of a line into two segments such that the ratio of the shorter segment to the longer segment equals the ratio of the longer segment to the whole line. This can be expressed by the quadratic equa-

tion $x^2 = x + 1$, the exact solution to which is $(1 \pm \sqrt{5})/2$, with its positive solution being an irrational number designated as ϕ (for Phidias, 1.62 to two decimals), the negative solution being -0.62 (ϕ '), and the negative of the negative solution being the golden section itself, approximately 0.62 (- ϕ ') (Green, 1995; Huntley, 1970; A. V. Voloshinov, 1996; please note that there are superficial notational differences among these three sources).

Within experimental aesthetics, starting with Fechner (1871, 1876), countless studies have attempted to examine the significance of the golden section more or less rigorously (for recent reviews, see Green, 1995; Hoege, 1995). However, most of these studies used (a) simple geometric shapes, especially lines and rectangles, as experimental stimuli, (b) presented the stimuli without any artistic or aesthetic context, and (c) relied on non-artists as research participants. For these and various other methodological and substantive reasons, the results remain inconclusive: When an entire recent issue of a major psycho-aesthetic journal was devoted to the golden section (Empirical Studies of the Arts, 1997, 15, No. 2), that seemed to be the consensus.

Even when the golden section was experimentally studied in an aesthetically more meaningful context (Konecni, 1997), so long as lay research participants and artificial stimuli continued to be used, it remained subtle and elusive, though detectable under certain specifiable conditions. However, when recently — within a single large experiment – the present author simultaneously engaged professional painters as research "informants" and participants, developed a research methodology suitable for them, and used authentic paintings as stimuli, the considerable significance of the golden section could be convincingly demonstrated (Konecni, 2000).

There is a twofold relevance for the present research of the last-mentioned experiment: It lies in the use of genuine paintings as stimuli and in the close scrutiny and measurement of the paintings' structure and content with regard to the various ways in which the golden section (0.62) and other proportions with classical and contemporary theoretical significance (i.e., 0.50, 0.67, 0.75, 0.80, 1.00) appear in them (on the significance of these other proportions, see Arnheim, 1966; Berlyne, 1971; Boselie, 1997; Shortess, Clarke, & Shannon, 1997). However, what was in the previous study a methodological innovation that helped in the investigation of the painters' (research participants') cognitive processes regarding the golden section, now becomes the research focus: Is the golden section used in 20th-century paintings and if so, how? Which other proportions tend to be used in those paintings which contain the golden section(s) and how?

The 20th-century is of particular interest with regard to the golden section because this period has been conspicuous, on one hand, for the alleged rejection of traditional learning and ideas, and, on the other, for geometricity and abstract conceptualizations.

This investigation appears to be the first in the literature to identify and measure systematically the occurrence of the golden section and other proportions within the

composition and structural elements (in addition to picture dimensions) of a relatively large number of very diverse paintings.

METHOD

The Sample of Paintings

The intention of this exploratory research obviously was not to obtain the accurate incidence and prevalence values (in epidemiological terms) of the golden section in the total population of 20th-century paintings (an impossible task of induction) or even in a genuine random sample (because various insurmountable logistical problems precluded such sampling). Rather, the goal was, first, to derive from a larger pool of paintings (of unknown size before the research was started) a sizable sample of paintings (about 100), by numerous painters, and with most of the decades represented, in which each painting would contain at least one golden section. (The discovery of such, "golden," paintings is a worthwhile goal in itself and may help future research.) Second, these paintings were to be closely scrutinized and measured with regard to other significant proportions in addition to the golden section. In other words, given that a painting contains a golden section, what other proportions is it likely to contain as well? How have the golden section(s) and these other proportions been used by the artists? (The important issue of the artists' conscious intention in the use of the golden section is not addressed in the present article, but see Konecni, 2000).

The initial search, by research assistants, was quasi-random and dictated, in part, by accessibility in various university and public art libraries in Southern California. The assistants were chosen intentionally for not being specialists in the visual arts and they were unaware of the overarching goals of the research. They were instructed to select volumes of color reproductions in as unsystematic a manner as possible (with regard to personal taste and familiarity, salience, alphabetical order, etc.). In this manner, a pool of some 250 color reproductions (of a size as close as possible to 11 X 8.5 inches or 27.94 X 21.59 cm) of 20th-century paintings was formed.

Once that good-quality copies of the reproductions were made, the research assistants were given specific guidelines about the type of elements and relationships within paintings that they should measure, in addition to picture dimensions (the guidelines are specified below). The measurements were made with extreme care, to the nearest millimeter, and repeatedly checked.

As it turned out, this initial pool of 250 paintings was sufficient for the detection of 95 paintings, each of which contained at least one instance of the golden section — which by itself is roughly informative about the incidence of the golden section in 20th-century paintings. The 95 works in the final sample (see Appendix) were by 52 painters and all the decades of the 20th-century were represented to some extent.

Note that the final sample was intentionally formed with a minimum of substantive intervention from the author, which presumably minimized the influence of any bias concerning the presence of other theoretically significant proportions in these, by definition golden, paintings.

Measurements

In the case of most paintings, including abstract ones, one can identify in their content many relationships that can be expressed as proportions. In the present work, only the major and prominent structural and compositional elements, on which there was general agreement among the author, art authorities, and colleagues, were selected for measurement. The following elements were measured in each of the 95 works, with the measurements described under (b), (c), and (d) representing the new substantive contribution of the present study to the golden-section literature.

- (a) Overall dimensions of the painting ("picture size"), which were converted into a ratio of the shorter side to the longer one, noting whether the painting was in the "portrait" or "landscape" orientation. Note that picture size was not of primary concern in the present research; rather, measuring picture size provided a criterion on which the present sample could be readily compared to larger samples described in the literature.
- (b) Vertical bi-section of the painting, which addressed the issue of left-right balance or the horizontal distribution of perceptual weights. When balance is discussed in art theory and psycho-aesthetics, this is the type of balance usually in question (e.g., Arnheim, 1974; Locher, 1996; Locher & Nagy, 1996; McManus, Edmondson, & Rodger, 1985). The point on the horizontal side of the painting where the vertical axis intersected it was expressed as a proportion of the length of the longer section to the entire width of the painting (bi-section at midpoint would thus equal 0.50 and as the difference between the two sections increased, the proportion would move in the direction of 1.00).
- (c) Horizontal bi-section, which addressed top-bottom balance or the vertical distribution of weights. The point on the vertical side where the horizontal axis intersected it was expressed as a proportion of the length of the longer section to the entire height with computational implications analogous to those in (b).

Whereas the relationships of the horizontal bi-section to the painter's vantage point, the "horizon line," or the single vanishing point of Brunelleschi and Masaccio, have obviously been seldom at issue in the past 125 years, the basic top-bottom distribution of weights remains of interest in studying composition (cf. Zettl, 1990, who suggests that the horizon line can be effectively placed at the golden section, but note that the horizon line thus placed may or may not be the horizontal bi-secting axis).

(d) In addition to the proportions obtained by line bi-section, as in (b) and (c) above, various proportions, including the golden section, were identified and measured in

geometric shapes, such as rectangles (including the "golden rectangle," where the ratio of the shorter to the longer side is 0.62), squares (the ratio of the sides = 1.00), circles (the ratio of perpendicular diameters = 1.00), as well as triangles, ellipses, and so on. Such shapes may occur in paintings both as pure forms and incorporated in the forms of the depicted objects. Therefore, the facial and bodily proportions of the key human figures were identified and measured, as well as the dimensions of the various structurally prominent objects, such as houses, bridges, windows, crosses, vases, chairs, plates, etc. Finally, the major compositional relationships, including the spatial arrangement of color (for example, in "color-field" paintings), were identified and measured.

RESULTS AND DISCUSSION

0.96-1.00

Picture Dimensions

The ratios of the shorter to the longer dimension are presented in the Appendix for all 95 paintings, along with their portrait/landscape orientations. The frequency with which the various ratios occurred, by picture orientation, is presented in Table 1.

Ratio	Frequency			
	"Portrait"	"Landscape"	Other	Total
0.49-0.55	1	3		4
0.56-0.60	2	1		3
0.61-0.65	10	10		20
0.66-0.70	7	5		12
0.71-0.75	12	6		18
0.76-0.80	10	3		13
0.81-0.85	7	5		12
0.86-0.90	3	0	ellipse (1)	4
0.91-0.95	2	0		2

TABLE 1. The Frequency of Ratios of Picture Dimensions by Picture Orientation

"square" (7)

Given the method of selection of the paintings for inclusion in the research sample, it is not surprising that the region in which the golden section falls (0.61-0.65) was encountered with the greatest frequency (20, or 21% of the sample). In fact, one might be surprised that it was not encountered more often. While the research assistants were not required to include in the final sample each of the works in the initial pool of 250 with the golden-section picture dimensions, it is likely that they would have done so, given the basic criterion for inclusion. One could therefore ten-

^{*} Ratios are of the shorter sides of paintings divided by longer sides. The seven paintings in the 0.96-1.00 region are squares or near-squares. Sample size is 95. The distributions of the "portrait" and "landscape" orientations did not significantly differ from each other (negligible χ^2).

tatively conclude that the golden picture dimensions are quite rare (20 of 250, 8%). As it turns out, this is in quite good agreement with the findings of the study by Shortess, et al. (1997, Figure 1, p. 169), in which the frequency of paintings with dimensions in the golden-section region can be estimated at 6%. The Shortess et al. study looked at 594 works painted after 1425, with a good 20th-century representation; 35 works in their sample (22 "portrait" and 13 "landscape") had picture dimensions in the golden-section region.

In the present research sample of 95, the next most frequently encountered regions were 0.71-0.75 (18, 19%) and 0.76-0.80 (13, 14%). Given the intentionally high number of golden-section picture proportions in the sample, the frequency of encountering the region around 0.75 was almost certainly artificially deflated and the general incidence of 3/4 for picture dimensions must be the highest of all. Such a conclusion is indeed borne out by both Fechner's (1876) and more recent findings (Shortess et al., 1997),² though French paintings, 1600-1900, may or may not be somewhat of an exception, depending on how they are thematically classified (Petrov, 1996).

Although the ratios of the overall picture dimensions were of secondary interest in the present research, it is reassuring that the sample is in agreement with the literature with regard to both the scarcity of the golden section and the high incidence of 0.75.

Vertical Bi-Section

Mirror-image symmetry with regard to a vertical line that intersects the width of the painting at midpoint (expressed as 0.50 in the present research, but as 1.00 if the ratio of one of the two sections to the other is the unit of analysis) had gradually begun to disappear from Western painting some 650 years ago. It was, of course, replaced by balance, or the harmonious left-right distribution of perceptual weights — but with reference to an identically (centrally) placed vertical axis. What art judgment tests and psycho-aestheticians alike mean by imbalance is the situation where the weights within a picture (or any visual display) deviate from a harmonious distribution around the central vertical axis (cf. Locher & Nagy, 1996, and the art tests they used as sources of stimulus materials).

However, note that almost every painting thus unbalanced can in fact be considered balanced,³ but with reference to an imaginary vertical axis that is shifted from the center.⁴ The present research was concerned with the existence and the degree of such shifts. Specifically, when an artist, consciously or not, chooses to break the norm — presumably consisting of the central placement of the vertically bi-secting line — where does that artist shift laterally and why?

For 58 of the 95 paintings in the sample, the vertical bi-section was found to be exactly at midpoint and for another five it was in the 0.501-0.545 region: There was always good agreement among the research assistants and the author with regard to

such placements and the reliability of these measurements is very high, 0.82 - 0.88. Thus, even in the 20^{th} -century, at least as it is represented in this sample, 66% of the paintings are balanced with regard to a centrally placed vertical axis and thus intuitively follow a traditional prescription.

Of the remaining 32 paintings, the vertical axis was shifted from the center to the 0.546-0.595 region in the case of 10 paintings and to the golden-section region (defined as 0.596-0.645 – a region well-established in the literature, see Hoege, 1995, and Konecni, 1997, 2000) in the remaining 22 paintings. The actual range for these 22 paintings was 0.60-0.64, with eight vertical bi-sections at 0.62.

When it is considered that to be included in the sample it was sufficient for a painting to contain a single instance of a golden section, and that there were many different ways for this criterion to be satisfied [see (a) through (d) under Measurements in the Method section], it follows that the number of the golden vertical bi-sections, while presumably inflated, is nevertheless impressive (23% of the sample and 69% of the non-center sub-sample). At the very least, it would appear that the shift of the vertical axis from the midpoint does not simply taper off, but rather that a jump is made from the center to the golden section via a relatively little-used fuzzy region in-between. And it is noteworthy that none of the 95 paintings had the vertically bi-secting line shifted to beyond the golden-section region.

Consciously or not, artists who avoided the (boring?) centrally placed vertical bisection, also avoided the (fuzzy and thus annoying?) adjoining region of the perceptible, but too small a shift from the central axis. The golden section may be the just-right region between the midpoint and two-thirds,⁵ in part because its legendary mathematical appeal gives it an intriguing advantage over, say, the nondescript 0.585. Left-right imbalance greater than a vertical axis at 0.64 apparently seemed unpalatable to the artists in this sample.

On the basis of these data, it is impossible to distinguish between the painters' habits acquired in formal training, their conceptual intent, and the spontaneously pleasing appearance of the golden vertical bi-section (to either the painters themselves and/or other viewers), as possible explanations of the shift to the 0.62 region. These possibilities and their combinations are extremely difficult to tease apart in a satisfactory experimental manner.

Horizontal Bi-Section

Balance with regard to a horizontal axis has been far less discussed, in large part because the strict top-bottom mirror symmetry has been so very rare even early in Western art (cf. Bouleau, 1963). But this type of balance, specifically in terms of a centrally positioned horizontal axis, certainly was quite common in the 15th- and 16th-Century Italian, Flemish, and Spanish religious art. Painters tended to fill up the canvass, typically with the bottom part being taken by earth-bound doings of interest to the Roman Church, with the top reserved for the godly, angelic, and sain-

tly presences, ascents, or (temporary) descents. A good example is El Greco's stupendous *The Burial of Count Orgaz* (1586), a painting of irregular shape, but the horizontal axis of which can reasonably be estimated to intersect the height at 0.55 (the longer, top, section divided by the entire height). Perfect overall balance is achieved by the lower-than-midpoint intersection being offset by the oval (and therefore smaller) area of the top one-third of the painting.

Even without statistical evidence, one could vouch that in the later centuries the top part became even longer — mostly by virtue of the painters relieving the sky of heavenly weight (in addition to the lack of the type of compensation by means of the painting's shape that is found in *Count Orgaz*). Already in Velasquez's *Las Meninas* (1656), within — by the present sample's standard — a very rare ratio of overall dimensions of 0.87 (317.50 X 275.60), the ratio analogous to that computed for *Count Orgaz* can be estimated at 0.63 (close to the lower golden section).

One could have reasonably expected the 20th-century "sky" to fill up, starting with the Cubists, and continuing with the various forms of abstraction, thus bringing the ratio close to 0.50. Indeed, in the present research, 62 of the 95 paintings were found to have the horizontal axis at 0.50 and another eight in the 0.501-0.545 region. At least in the present sample, there is a very considerable degree (74%) of safe top-bottom balance with regard to a centrally placed horizontal axis. For the majority of the painters represented in this sample, the top-bottom direction seems to have been a somewhat less attractive one for experimentation with balance than the left-right: It may be perceptually or aesthetically a less hospitable medium.

The remaining 25 paintings were almost equally divided between the regions of 0.546-0.595 (13 paintings) and 0.596-0.645 (the golden-section region with 12 paintings, all within the range of 0.60-0.63, five at 0.62). Thus, for none of the 95 paintings did the ratio of the longer part to the entire height, resulting from the horizontal bi-section, exceed 0.63. The golden-section region was again the limit of the off-center balancing experimentation.

In which direction, up or down, does the shift of the horizontal bi-secting axis occur in these 25 works? There is a somewhat greater tendency for the intersect to be above the midpoint (15 of 25) and the majority of these 15 (nine) are paintings with the horizontal bi-section in the golden-section region [however, the relevant statistic $\chi^2(1) = 2.07$, p < .20 is not statistically significant at an acceptable level].

To summarize, the horizontal bi-section data suggest that when artists choose to go off-center in terms of the horizontal bi-section, the entire 0.501-0.63 region is evenly represented. In addition, the placement into the upper golden-section region is chiefly responsible for the general, but statistically non-significant, tendency to move the horizontal axis upward.

Double Shift into the Golden Section

In Table 2 is the summary of the combined results for the vertical and horizontal bi-

section in all 95 paintings. The paintings are obviously very unevenly distributed and four of the nine cells have entries of five or less (which precludes the χ^2 , in any case unnecessary, from being legitimately computed). Over half of the paintings in the sample (49) are in the cell where both the vertical and the horizontal bi-section are at, or very close, to 0.50.

Horizontal Bi-Section		Vertical Bi-Section	
	0.50-0.545	0.546-0.595	0.596-0.645
	3, 5, 6, 8, 10,	20, 25, 48,	4, 13, 22,
	11, 12, 14, 15, 16,	49, 51, 60,	28, 32, 40,
	17, 18, 19, 23, 24,	63, 93.	43, 50, 62,
	26, 29, 30, 33, 35,		71, 73, 74,
	36, 37, 39, 41, 44,		94.
0.50-0.545	45, 46, 47, 53, 54,		
	59, 61, 65, 66, 67,		
	68, 69, 70, 72, 75,		
	80, 82, 83, 84, 86,		
	88, 89, 92, 95.		
	49	8	13
	9, 27, 31, 57, 78,	38, 79.	2, 21.
0.546-0.595	81, 85, 90, 91.		
	9	2	2
	34, 42, 76, 77, 87		1, 7,52,
0.596-0.645			55, 56, 58,
	5	0	64. 7

Numbers in the body of the table are those assigned to the paintings in the Appendix.
 Totals in each of the nine cells are in large bold and add up to 95. See Ref. [15] about No. 73.

The pattern of shifts away from the double central balance is highly instructive and easy to grasp in Table 2. For the present purpose, however, the most interesting cell is undoubtedly the one with paintings which are most removed from both central axes, that is, those shifted into the golden-section region with regard to both types of bi-section.

There were seven paintings in this cell — a Bakst, an Ernst, two Licinis, a Nicholson white relief, a Roerich, and a Vuillard (see Table 2 and the Appendix for details). Without judging other attributes of these paintings, perhaps only the Vuillard strikes one as having a dubious composition. In all the other cases, the double displacement is effectively used to maximize both interest and focus on the desired feature. Without exception, not counting the Vuillard, the key structural and thematic features would have been boring or awkward had they been more centrally placed with regard to either axis. These painters mainly sought the viewer's interest, not pleasure.⁶

The Golden Section Within Paintings

Table 3 presents the occurrences of the golden section within paintings (excluding both types of bi-section and also the overall picture dimensions). The table shows the number of paintings in which the golden section occurred once, twice, and so on. respectively, in the structural and compositional elements mentioned in the Method section. In addition, for comparison, the average (per painting) and the total number of occurrences of other significant proportions are given for sub-group of paintings. Although the method of selection clearly contributed considerably to the fact that the golden section was used 254 times in these 88 paintings (no occurrences within seven), or 2.89 per painting, this high number testifies to the popularity of use of the golden section in the structural, compositional, and thematic elements of numerous 20th-century works. The variety of styles and decades in which such heavy use occurred is impressive and best demonstrated by mentioning the six paintings in Column B of Table 3, in each of which the golden section was used over five times: Delaunay (Appendix No. 8, 1912), Macke (17, 1914), Klee (35, 1922), Lohse (67, 1943-68), Matisse (69, 1946), and Gorbachev (94, 1998). In most, but not all of these paintings, repeated geometric patterns were used. Given the preponderance of busy patterns that entirely fill the canvasses of these works, it is not surprising, as can be seen in Table 2, that 11 of 12 vertical and horizontal bi-sections of these six works were in the 0.50 region.

TABLE 3.	. The Golden	Section in th	e Structura	Lements	of Paintings*

Α	В	С
The Number of	The Number of Paintings	The Mean and Total Number of
Golden Sections	Characterized by the	Other Proportions in the
in a Painting	Entries in Column A	Entries of Column B
0	7	1.57 (11)
1	20	0.70 (14)
2	24	1.08 (26)
3	19	0.63 (12)
4	9	1.56 (14)
5	10	2.70 (27)
> 5	6	3.00 (18)

In Column A, the numbers do not include the golden ratios of the overall picture dimensions and those for both the vertical and horizontal bi-sections. Entries in Column B add up to 95. In Column C, "other" proportions are 0.50, 0.55, 0.67, 0.70, 0.75, 0.80, 1.00, or, rather, regions consisting of 0.025 on both sides of each proportion (but only 0.015 in the region between 0.67 and 0.70) The total number of "other" proportions in Column C is 122.

Column C in Table 3 shows that other proportions are used in key structural elements a fair amount (1.57 per painting) in those (seven) works in which the golden section does not appear — and considerably more than that in the paintings in which the golden section makes a moderate appearance (one, two, and three occurrences). It is only in paintings with four and more occurrences of the golden section that the average use of other proportions equals and then substantially exceeds that in the seven paintings without golden sections.

So, two tendencies seem to be at work. One is to avoid the golden section altogether and use other proportions (these seven paintings were included in the sample because of the ratios of overall dimensions or of the bi-sections). The other is a preference for geometricity that is reflected in the clear trend that as the use of the golden section increases in frequency so does that of the other proportions.

Table 4 shows which those other proportions are. The total number of occurrences of non-golden-section proportions in the 95 paintings was 122 (1.28 per painting), of which over a half (63, 52%) were 1.00 and 0.67. The ratio of 1.00 was especially frequently used — in squares, partial squares, implied perpendicular diameters of circles, the shapes of regions around key figures, and within various depicted objects. This finding confirms 1.00 as a serious rival to the golden section (Konecni, 2000).

Proportion	The Number	er of Occurr	ences of a l	Proportion i	n a Painting
	0	1	2	3	6
0.50	80	10	4	1	
0.55	86	8	1		
0.67	76	16	1	2	
0.70	86	7	1	1	
0.75	91	4			
0.80	83	12			
1.00	67	23	2	2	1

TABLE 4. The Number of Paintings with Non-Golden-Section Proportions in Their Structural Elements*

The paintings with the greatest number of occurrences of 1.00, according to Table 4, are another Klee (No. 46, 1926), another Nicholson relief (66, 1943), an aleatory Arp collage 22, 1916), a Duncan (59, 1935?), and the previously-mentioned Lohse (67, 1943-68) that also used many golden sections. Whereas the Nicholson, Arp, and Lohse are geometric abstractions, the 1.00 proportion is incorporated into human heads and bodily shapes in the Duncan. In the Klee, abstract shapes (including six circles) float around two realistic fish, which are on an elliptoid plate of golden-section proportions.

CONCLUSIONS

Symmetry, in its different forms and modifications, is one of the fundamental principles of art (and life, and perhaps all nature). In developing further Weyl's (1952) seminal ideas on symmetry, Voloshinov (1996, p. 111) stated: "The golden section is a symmetry of similarity of the parts and the whole." It is instructive to examine the conclusions that can be drawn from the present research in the light of Voloshinov's statement.

1. The golden ratio of the overall picture dimensions is rare and thus a poor predic-

^{*} The ratios of the overall picture dimensions and those for both the vertical and horizontal bi-sections are not included. All rows add up to 95. The total number of occurrences of non-golden-section proportions in the 95 paintings was 122 (or 1.28 per painting).

tor of the probability of occurrence(s) of the golden section in a painting's balance lines or structural elements.

- 2. Central balance with regard to both the vertical and horizontal axes has remained dominant throughout the 20th-century. However, when the vertical axis is shifted laterally, it is most often to the golden section region. The up-down shift, when it occurs, is evenly distributed across the entire region between the midpoint and the golden section. For both balance directions, the golden section region is the limit of the shift. The double shift into the golden section region is rare, but informative. An important function of the golden section may be that it introduces an element of tension, complexity, and risk.
- 3. It is very common for the golden section to be used within paintings in the construction of human faces, bodies, and enveloping areas, various objects (such as crosses), and also of geometric forms and color fields of various degrees of abstraction. The ratio of 1.00 is the main "rival."
- 4. In elements within paintings, the non-golden-section proportions were used most often in the works in which the golden section (a) did not occur at all, and (b) occurred a great deal. The relative presence of the golden and other proportions seems to be affected by several structural and stylistic strategies.

With regard to 1., Voloshinov's statement appears to be wrong. As for 3. and 4., it is insufficiently detailed to be useful. And with regard to 2., it is correct, especially when one keeps in mind that for the bi-sections, the golden section was computed as a ratio of the longer section to the entire length. Such is perhaps the fate of most plausible general statements.

Footnotes

- 1. For reprints please write to V. J. Konecni, Department of Psychology, University of California, San Diego, La Jolla, California 92093-0109. A brief version of this paper was presented by invitation at the international conference "20th-Century Art: Traditions, Achievements, Innovations" held at the Hermitage Museum, St. Petersburg, Russia, December 1999. I am very grateful to my student-assistants, but especially to Laney Cline (an alumna of the University of California, San Diego, and now a Columbia University graduate student) for her competent and tireless help with this research.
- 2. Fechner (1876) computed the ratios of picture dimensions (longer side by shorter side) for some 1,400 works dated prior to about 1860 and obtained a mean of about 1.30 the inverse of which, obtained by computing the shorter-side by longer-side ratios, as in the present research, is 0.77. In the cited study by Shortess et al. (1997), the median for 594 works (283 "portrait," 301 "landscape," 10 squares) was 1.31, the inverse of which is 0.76. Furthermore, the approximate 4/3 (3/4) median ratio also held for the 20th-century works.
- 3. A striking example of a special case is Barnett Newman's *Tundra* (1950, No. 73 in the sample). This is a monochrome field painting that is fully bi-sected by a black vertical line parallel to the sides. Interestingly, the line bi-sects the painting's width at the golden section. *Tundra* is thus highly unbalanced and cannot be balanced with regard to a vertical axis, because there is no weight to counter that of the painted black line. Presumably the golden bi-section of the painting's

horizontal side reduces the perceptual/aesthetic discomfort. (Incidentally, the horizontal balance axis intersects the vertical side at 0.50.)

- 4. See McManus et al. (1985), whose Figure 2, p. 315, nicely illustrates the issues under consideration: Paul Nash's *Totes Meer* (*Dead Sea*, 1940-41) is an unbalanced painting in traditional terms (more weight on the left, due mostly to a large sun or moon) and the subjects counter this by placing the balance fulcrum to the left of the center. When 19% at the left edge of the painting is chopped off, but without removing the critical celestial body, which thus acquires still greater weight, the subjects place the balance fulcrum comparatively even further to the left of the midpoint (of the modified stimulus). See also Locher & Nagy (1996), whose Figure 2, p. 22, shows a pair of art-test pictures that are identical except for the left-right placement of a human figure in the foreground. The unbalanced picture (with regard to the vertical central axis) can be considered balanced if the vertical axis is shifted from the midpoint to as it happens, according to my measurements the (right-hand) golden-section region.
- 5. Various forms of this idea have, of course, been expressed before (e.g., Arnheim, 1966; Berlyne, 1971; McManus & Weatherby, 1997), but the idea has a much longer history which can be traced in modern times to German experimental psychologists Wundt, Fechner, Külpe, and Lipps at the end of the 19th-Century.
- 6. Pleasingness and interestingness have been considered the main evaluative dimensions in psycho-aesthetic research (e.g., Berlyne, 1971; Sargent-Pollock & Konecni, 1977).

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Appendix

The Sample of 95 Paintings

No.	Painter	Painting	Year	P/L*	Ratio
1.	Jean Vuillard	Mother and Child	1900	"Square"	0.96
2.	Lyov Bakst	The Beautiful Harem Girl	1910	P	0.61
3.	Oskar Kokoschka	Paul Scheerbart	1910	P	0.67
4.	Piet Mondrian	Church Tower at Domburg	1910-11	P	0.66
5.	Jean Vuillard	Mme Hessel Seated	1910-12	P	0.78
6.	Lyov Bakst	The Punishment of Cupid	1910?	Ellipse	0.89
u.	Ejov Bakst				
			4010	_	0.45
7.	Lyov Bakst	Sketch of a Woman's Dress	1912	P	0.65
8.	Robert Delaunay	Window on the City	1912	P	0.82
9.	André Derain	The Calvary	1912	P	0.86
	Fernand Léger	Paris Seen Through a Window	1912	P P	0.74 0.76
	Kazimir Malevich	The Woodcutter	1912	P P	0.73
	Pablo Picasso	The Violin	1913	P P	0.73
	Giorgio de Chirico	The Song of Love	1914	P	0.90
	Paul Klée	Red and White Domes	1914	P	0.90
	August Macke	Vendor With Pitchers	1914 1914	L	0.77
	August Macke	Man on Donkey	1914	P	0.87
	August Macke	Bright House	1914	P	0.80
	Kazimir Malevich	Composition with Mona Lisa	1914	P	0.71
	Piet Mondrian	Blue Façade	1915	P	0.71
	Kazimir Malevich	Supremacist Painting. Rectangle and Circle	1916	P	0.86
	Henri Matisse	The Piano Lesson	1916	P	0.49
	Jean Arp	Rectangles Arranged According to the Laws of Chance	1917	P	0.74
	Amadeo Modigliani	Girl with Braids	1917	P	0.73
	Georges Rouault	The Old Clown	1917?	į.	0.75
	John Duncan	The Kelpie	1918	P	0.64
	Juan Gris	Portrait of Mme Lipshutz Reclining Nude	1919	i	0.62
	Amadeo Modigliani	Portrait of Lunia Czechowska	1919	P	0.70
	Amadeo Modigliani John Duncan	Baba & Billy	1920	P.	0.75
	Max Ernst	Dada Gauguin	1920	Ĺ	0.73
	Kazimir Malevich	Supremacist Painting (Large Cross on White)	1920-27	P	0.83
51.	Razinii Malevicii	Supremiers Funding (Eurge Cross on Winter,			
32.	Pierre Bonnard	The Open Window	1921	P	0.81
33.	Piet Mondrian	Composition	1921	"Square"	1.00
34.	Paul Klée	Senecio	1922	P	0.94
35.	Paul Klée	Ambassador of Autumn	1922	L	0.81
36.	Oskar Kokoschka	Self-Portrait at the Easel	1922	P	0.61
37.	Oskar Kokoschka	Self-Portrait with Crossed Arms	1923	P	0.64
38.	Cristóbal Ruiz	Portrait of His Daughter in a Corridor	1923	Р	0.71
39.	Cristóbal Ruiz	Interior	1923?	"Square"	0.97
40.	Fernand Léger	The Syphon	1924	P	0.72
41.	Fernand Léger	Mechanical Element	1924	L	0.75
42.	Joan Miró	Spanish Dancer	1924	P	0.62
43.	Salvador Dalí	Girl's Back	1925	P	0.71
44.	Theo van Doesburg	Counter-Composition of Dissonants XVI	1925	L	0.56
45.	Franz Seiwert	After Work	1925	P	0.70
46.	Paul Klée	Around the Fish	1926	L	0.72
	Kurt Schwitters	Merz 48: Berlin	1926	P	0.74
48.	Pablo Picasso	The Studio	1927-28	L	0.63
49.	Paul Klée	Prince	1930	Р	0.67
50	Vassiliy Kandinsky	Suite No. 424	1931	L	0.64
	Pablo Picasso	Pitcher and Bowl of Fruit	1931	L.	0.80
	Nikolay Roerich	Zoroaster	1931	L	0.63

4. Pablo Picasso Wornan Writing 1932 P 1935 L 1935			•			
55. Osvaldo Licini Rhythm 1933 L 0 56. Osvaldo Licini Windmill 1935 L 0 57. Bruno Munari Even the Frame 1935 L 0 58. Ben Nicholson White Relief 1935 L 0 69. John Duncan Phlegethon 1935? L 0 60. Georges Rouault The Old King 1937.7 P 0 61. Morris Hirshfield Lion 1937.39 P 0 62. Morris Hirshfield Lion 1939 L 0 63. Victor Brauner Nude and Spectral Still Life 1939 P 0 64. Max Ernst Napoleon in the Wilderness 1941 P 0 65. Piet Mondrian New York City 1942 "Square" 1 65. Piet Mondrian New York City 1943 L 0 66. Ben Nicholson Seven Pillars of Wisdom 1943 L 0 67. Richard Lohse Fifteen Systematic 1946 P	53.	Pierre Bonnard	White Interior	1932	L	0.70
55. Devaldo Licini Windmil 1935 L 0.0	54.	Pablo Picasso	Woman Writing	1932	P	0.63
Sear Nicholson	55.	Osvaldo Licini	Rhythm	1933	L	0.74
See Deal Nicholson White Relief 1935 L 0 0	56 .	Osvaldo Licini	Windmill	1935	L	0.76
59. John Duncan Phlegethon 1935? L 0 60. Georges Rouault The Old King 1937.7 P 0 61. Morris Hirshfield Lion 1937.39 P 0 63. Victor Brauner Nude and Spectral Still Life 1939 L 0 64. Max Ernst Napoleon in the Wilderness 1941 P 0 65. Piet Mondrian New York City 1942 "Square" Incholson 1943 L 0 67. Richard Lohse Fifteen Systematic 1943.6 "Square" Incholson 1943.6 "Square" Incholson "Square" Incholson 1943.6 "Square" Incholson 1943.6 "Square" Incholson Henri Matisse Polynesia, The Sea 1946 P 0 1946 P 0 0 1946 P 0 1948 P 0 0 1946 P 0 0 1946 P 0 0 1946 P 0 0 0 1946 P 0 0 0 0 0 <td>57.</td> <td>Bruno Munari</td> <td>Even the Frame</td> <td></td> <td></td> <td>00.1</td>	57.	Bruno Munari	Even the Frame			00.1
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76. René Magritte Promenades of Euclid 1955 P 0 77. Arnulf Rainer Cross 1956 P 0 78. Arnulf Rainer Cross 1956 P 0 79. Andrew Wyeth Chambered Nautilus 1956 L 0 80. Mark Rothko Light Red over Black 1957 P 0 81. Patrick Heron Horizontal Stripe Painting: November 1957–January 1958 1957-58 P 0 82. Jasper Johns Three Flags 1958 L 0 83. Arnulf Rainer Sea 1959-60 L 0 84. Arnulf Rainer Orange Overpainting on Yellow on White 1961 L 0 85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1	75	Mark Rothko	No. 8	1952	Þ	0.85
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80. Mark Rothko Light Red over Black 1957 P 0 81. Patrick Heron Horizontal Stripe Painting: November 1957–January 1958 1957–58 P 0 82. Jasper Johns Three Flags 1958 L 0 83. Arnulf Rainer Sea 1959–60 L 0 84. Arnulf Rainer Orange Overpainting on Yellow on White 1961 L 0 85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 89. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Pres					-	0.51
81. Patrick Heron Horizontal Stripe Painting: November 1957–January 1958 1957-58 P 0 82. Jasper Johns Three Flags 1958 L 0 83. Arnulf Rainer Sea 1959-60 L 0 84. Arnulf Rainer Orange Overpainting on Yellow on White 1961 P 0 85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry l 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" I 94. Yuriy Gorbachev Evolution I 1998 P 0						0.66
83. Arnulf Rainer Sea !959-60 L 0 84. Arnulf Rainer Orange Overpainting on Yellow on White 196! P 0 85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0			_			0.56
83. Arnulf Rainer Sea !959-60 L 0 84. Arnulf Rainer Orange Overpainting on Yellow on White 196! P 0 85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	82	lasper Johns	Three Flags	1958	L	0.68
84. Arnulf Rainer Orange Overpainting on Yellow on White 1961 P 00 85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0		•	•			0.65
85. Mark Rothko Horizontals, White over Darks 1961 L 0 86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	027			.,,,	_	0.05
86. Ellsworth Kelly Red Blue Green 1962 L 0 87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	84.	Arnulf Rainer	Orange Overpainting on Yellow on White	196!	P	0.65
87. Paul Delvaux The Mirage 1967 L 0 88. Bridget Riley Late Morning 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	85.	Mark Rothko		1961	i.	0.61
88. Bridget Riley Late Morning , 1967 L 0 89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	86.	Ellsworth Kelly	Red Blue Green	1962	L	0.54
89. Adnan Çoker Open Symmetry I 1975 L 0 90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	87.	Paul Delvaux	The Mirage	1967	L	0.62
90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0	88.	Bridget Riley	Late Morning	; 1967	L	0.63
90. Richard Diebenkorn Ocean Park No. 96 1977 P 0 91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution I 1998 P 0					_	
91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution 1 1998 P 0						0.75
91. Stephen McKenna Two Black Diamonds 1986 L 0 92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution 1 1998 P 0	90.	Richard Diebenkorn	Ocean Park No. 96	1977	Р	0.94
92. Peter Halley Red Cell 1988 L 0 93. Paula Rego Time — Past and Present 1990-91 "Square" 1 94. Yuriy Gorbachev Evolution 1 1998 P 0	91.	Stephen McKenna	Two Black Diamonds	1986	L	0.67
93. Paula Rego Time — Past and Present 1990-91 "Square" I 94. Yuriy Gorbachev Evolution I 1998 P 0						0.85
94. Yuriy Gorbachev Evolution 1 1998 P 0		•			_	1.00
					34000	
OS Viniu Cortactory Com						0.76
93. Furly Gordachev Sun 1998 P 0	95.	Yuriy Gorbachev	Sun	1998	P	0.75

^{*}Horizontal lines separate decades.

 $^{^{\}text{h}}$ P = "portrait" orientation; L = "landscape" orientation; "Square" = near-square. Ratio of the shorter to the longer picture dimension.